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An Exact Differential Method to Determine Liapunov Stability

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In this Note a simple method is presented to find the time derivative of a Liapunov function[1,2,3] in order to test the asymptotic stability of an autonomous system. The concepts[1,2,3] of Liapunov's function are not presented here. However, it may be mentioned that if the Liapunov function in the study of motion of a system is positive definite and its total rate of change with respect to time is negative definite or negative semidefinite, then the motion of the system is considered to be asymptotically stable.

The Liapunov function $V(x)$ is a scalar, and it is not unique. The line integral[4] between two points is independent of the path of integration, and it is assumed that the curl grad V is equal to zero.

In the analysis of the motion of a system, we obtain a system of equations of motion. Utilizing the state variable analysis,[2] we may reduce the equations of motion into a set of state variable equations. Without solving these equations, we may determine the asymptotic stability of the motion by employing the Liapunov stability criteria in the following formulations:

$$\text{Let } V = \int_{p_1}^{p_2} d[\Sigma x_i^2 x_j^2]; \quad i,j = 1,2,\ldots n \tag{1}$$

For brevity, we consider 3-dimensional case in Eq. (1) and obtain

$$V = \int_{p_1}^{p_2} [(4x_1^3 + 4x_1x_2^2 + 4x_1x_3^2)dx_1 + (4x_2^3 + 4x_2x_3^2 + 4x_2x_1^2)dx_2 + (4x_3^3 + 4x_3x_1^2 + 4x_3x_2^2)dx_3] \tag{2}$$

Recalling that the integral of a gradient is independent of the path of integration, we write

$$V = \int_{p_1}^{p_2} \left[\frac{\partial V}{\partial x_1} dx_1 + \frac{\partial V}{\partial x_2} dx_2 + \ldots + \frac{\partial V}{\partial x_n} dx_n\right] \tag{3}$$

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For 3-dimensional case

$$V = \int_{p_1}^{p_2} \left[\frac{\partial V}{\partial x_1} dx_1 + \frac{\partial V}{\partial x_2} dx_2 + \frac{\partial V}{\partial x_3} dx_3\right] \tag{4}$$

Using the exactness criteria, we get from Eqs. (2) and (4),

$$\begin{aligned} \partial V/\partial x_1 &= 4x_1^3 + 4x_1x_2^2 + 4x_1x_3^2 \\ \partial V/\partial x_2 &= 4x_2^3 + 4x_2x_3^2 + 4x_2x_1^2 \\ \partial V/\partial x_3 &= 4x_3^3 + 4x_3x_1^2 + 4x_3x_2^2 \end{aligned} \tag{5}$$

And the time derivative $\dot{V}(x)$ is

$$dV/dt = \Sigma(\partial V/\partial x_i)\dot{x}_i; \quad i = 1,2,\ldots n \tag{6}$$

The procedures are explained by an example.

Let us consider the system

$$\dot{x}_1 = -x_2 - x_1^3; \quad \dot{x}_2 = x_1 - x_2 \tag{7}$$

using Eqs. (5) and (6), we get for 2-dimensional case

$$\begin{aligned} dV/dt &= (4x_1^3 + 4x_1x_2^2)(-x_2 - x_1^3) + (4x_2^3 + 4x_2x_1^2)(x_1 - x_2) \\ &= -(4x_1^6 + 4x_2^4 + 4x_1^4x_2^2 + 4x_2^2x_1^2) \end{aligned}$$

which is negative definite. Therefore, the system [Eq. (7)] is asymptotically stable.

As far as line integral and the assumptions are concerned, this technique has similarity only with the existing variable-gradient method.[2] An iterative procedure is required to determine the coefficients of the gradient matrix in the variable-gradient method. Although the partial derivative of an arbitrary positive definite function $V(x)$ with respect to the state variables may lead to some results, the exactness approach in the field of curl grad $V = 0$ may establish a mathematical criteria for a methodology.

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Vibration Analysis by Differential Holographic Interferometry

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A NEW double-pulse method of holographic interferometry is proposed. This technique intends holography to permit measurements on large, noisy subjects vibrating to

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